4. V. K. Shchukin, Heat Exchange and Hydrodynamics of Internal Flows in Mass Force Fields [in Russian], Mashinostroenie, Moscow (1970).
5. Gebhart, Dring, and Polymeropulos, Teploperedacha, No. 1 (1967).
6. MacGregor and Emery, Teploperedacha, No. 3 (1969).
7. F. Kreis, in: Advances in Heat Transfer [Russian translation], Mir, Moscow (1971).
8. Vlit and Layou, Teploperedacha, No. 4 (1969).
9. Johnston, Teor. Osnovy Inzh. Raschet., No. 2 (1973).

EFFECT OF TEMPERATURE AND SLOPE OF A GAS MAIN IN NONISOTHERMAL UNSTEADY GAS MOVEMENT

L. E. Danielyan

UDC 533.6

The effect of the temperature and slope of a pipeline on the variation in gasdynamic parameters is studied using numerical methods.

The solution of the problem of the unsteady movement of a real gas through a long gas main has great theoretical and practical importance. The problem becomes relatively complicated when the nonisothermy and slope of the gas main are taken into account. The effect of the slope of the gas main on the parameters of the moving gas becomes important in mountainous terrain or when the gas is fed through a long gas main of large diameter with a slope which varies (even slightly) from horizontal.

Many reports have been devoted to these problems. Numerous linearized solutions of the problem of unsteady movement of a real gas through long gas mains are known ([1-9] and others).

The case of a vertical pipe set in natural soil, which occurs when gas is extracted from great depths, is of particular interest.

The unsteady nonisothermal movment of a gas through a long pipeline is examined below.

1. Differential Equations of Motion. Initial and Boundary Conditions

Let us consider the one-dimensional, unsteady, nonisothermal motion of a gas in a long gas main. Such gas motion can be described by the system of differential equations [1]

$$-\frac{\partial p}{\partial x}=\frac{\lambda\rho u^2}{8\delta}+\rho g\sin\alpha;$$

$$-\frac{\partial \rho}{\partial t}=\frac{\partial}{\partial x}(\rho u); \tag{1.1}$$

$$p=\rho gRT;\ G=gs\rho u.$$

It is assumed that the gas temperature varies along the gas main as a function of the coordinate and is a function assigned in advance.

Placing the origin of coordinates at the start of the pipe, directing the axis along the length of the pipe, and assuming for determinacy that the gas temperature along the pipeline varies by a linear law, one can express T(x) by the following equation [10]:

$$T(x)=T_s-(T_s-T_e)\frac{x}{L}, \tag{1.2}$$

Erevan Zootechnical-Veterinary Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 4, pp. 619-624, April, 1976. Original article submitted April 15, 1975.

where T_s is the gas temperature at the start of the pipe, T_e is the gas temperature at the end of the pipe, and L is the length of the pipeline.

Using obvious mathematical transformations, the system (1.1) can be reduced to the following nonlinear differential equation of second order relative to the square of the pressure:

$$\frac{\partial^2 P}{\partial x^2} = \sqrt{-\frac{\lambda}{4\delta g RT}\frac{1}{P}\frac{\partial P}{\partial x} - \frac{\lambda \sin\alpha}{2\,\delta g R^2 T}}\frac{\partial P}{\partial t} + \left(\frac{1}{T}\frac{\partial T}{\partial x} - \frac{2\sin\alpha}{RT}\right)\frac{\partial P}{\partial x} + \frac{4\sin\alpha}{RT^2}\frac{\partial T}{\partial x}P, \quad (1.3)$$

where $P(x, t) = p^2(x, t)$.

We assign the intial and boundary conditions:

$$\begin{gathered} P = P_s = \text{const} \text{ at } x = 0; \\ \frac{\partial P}{\partial x} = -\frac{\lambda R T_e}{4\,\delta g s^2} G^2(t) - \frac{P_e \sin\alpha}{R T_e} \text{ at } x = L; \\ P = P_0(x, \alpha) \text{ at } t = 0; \end{gathered} \quad (1.4)$$

Here G(t) is a given function characterizing the law of variation of the gas flow rate at the end of the pipeline; $P_0(x, \alpha)$ is a function showing the law of variation of the square of the pressure along the pipeline with a steady mode of nonisothermal motion, determined by the equation [11]

$$P_0(x, \alpha) = \frac{P_s(1+KL)^{-\frac{2g\sin\alpha}{RT_s K}} - P_e}{(1+KL)^{-\frac{2g\sin\alpha}{RT_s K}} - (1+KL)^2}(1+Kx)^2 - \frac{[P_s(1+KL)^2 - P_e](1+Kx)^{-\frac{2g\sin\alpha}{RT_s K}}}{(1+KL)^{-\frac{2g\sin\alpha}{RT_s K}} - (1+KL)^2}, \quad (1.5)$$

where P_s and P_e are the squares of the pressures at the start and end of the pipeline; $K = T_e - T_s/LT_s$.

By determining the pressure from Eq. (1.3), one can calculate the velocity, flow rate, and density of the gas from the equations

$$u(x, t) = \sqrt{-\frac{8\delta g R}{\lambda}\frac{T(x)}{p(x, t)}\frac{\partial p(x, t)}{\partial x} - \frac{8\delta g \sin\alpha}{\lambda}}; \quad (1.6)$$

$$G(x, t) = \frac{s}{RT(x)} p(x, t) u(x, t); \quad (1.7)$$

$$\rho(x, t) = \frac{1}{gRT(x)} p(x, t). \quad (1.8)$$

Thus, the problem is reduced to the solution of Eq. (1.3) with the boundary and initial conditions (1.4).

2. Solution of Stated Problem

Before proceeding to the solution of the problem, Eq. (1.3) and conditions (1.4) and (1.2) are preliminarily reduced to dimensionless form using the following relations:

$$P = P_s P'; \; x = Lx'; \; t = t_0 t'; \; T = T_s T'; \; G = G_0 G', \quad (2.1)$$

where the quantities with primes are dimensionless; P_s, L, t_0, G_0, and T_s are the characteristic pressure, length, time, flow rate, and temperature, respectively.

The gas flow rate in the steady mode of operation is taken as the characteristic flow rate, the square of the gas pressure at x = 0 for the steady mode (1.5) is taken as the characteristic pressure, the length of the gas main is taken as the characteristic length, and the gas temperature at x = 0 is taken as the characteristic temperature.

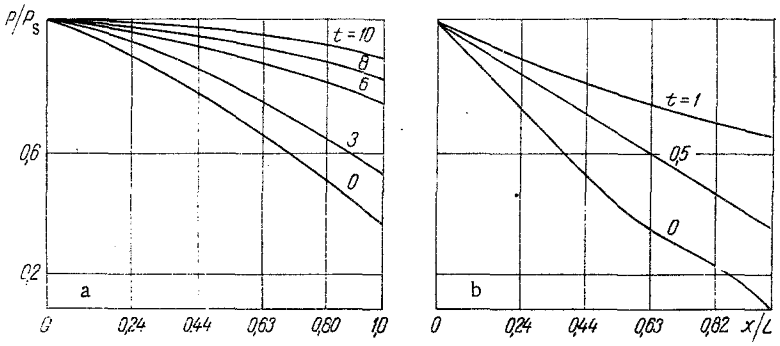

Fig. 1. Pressure distributions along gas main for separate times: a) with $\alpha = 0$; b) with $\alpha = \pi/2$.

The characteristic time is determined from Eq. (1.3):

$$t_0 = \sqrt{\frac{\lambda L^3}{4\,\delta g R T_s}}. \qquad (2.2)$$

In dimensionless variables Eq. (1.3) and the boundary and initial conditions (1.4) and (1.2) take the form

$$\frac{\partial^2 P}{\partial x^2} = \sqrt{-\frac{1}{TP}\frac{\partial P}{\partial x} - \frac{2L\sin\alpha}{RT_s T^2}}\,\frac{\partial P}{\partial t} + \left(\frac{1}{T}\frac{\partial T}{\partial x} - \frac{2L\sin\alpha}{RT_s T}\right)\frac{\partial P}{\partial x} + \frac{4L\sin\alpha}{RT_s T^2}\frac{\partial T}{\partial x}P, \qquad (2.3)$$

$$P = 1 \quad \text{at} \quad x = 0;$$

$$\frac{\partial P}{\partial x} = AG^2(t) - \frac{2LP_e\sin\alpha}{P_s RT_e} \quad \text{at} \quad x = 1; \qquad (2.4)$$

$$P = P_0(x, \alpha),\ T = 1 - \left(1 - \frac{T_e}{T_s}\right)x \quad \text{at} \quad t = 0. \qquad (2.5)$$

where $A = -(\lambda RT_e LG_0^2/4\delta gP_s s^2)$. The primes are omitted for convenience of notation.

We can represent Eq. (2.3) in the form

$$\frac{\partial P}{\partial t} = \frac{1}{M}\frac{\partial^2 P}{\partial x^2} + \frac{C_1 + C_2\sin\alpha}{MT}\frac{\partial P}{\partial x} + \frac{2C_1C_2\sin\alpha}{MT^2}P, \qquad (2.6)$$

where

$$-C_1 = \frac{\partial T}{\partial x};\quad C_2 = \frac{2L}{RT_s};\quad M = \frac{1}{T}\sqrt{-\frac{T}{P}\frac{\partial P}{\partial x} - C_2\sin\alpha}.$$

Defining

$$\frac{C_1 + C_2\sin\alpha}{MT} = -N;\quad \frac{2C_1C_2\sin\alpha}{MT^2} = -H,$$

we obtain

$$\frac{\partial P}{\partial t} = \frac{1}{M}\frac{\partial^2 P}{\partial x^2} - N\frac{\partial P}{\partial x} - HP. \qquad (2.7)$$

In [12] the model equation

$$\frac{\partial \Phi}{\partial t} = \frac{1}{M}\frac{\partial^2 \Phi}{\partial x^2} - N\frac{\partial \Phi}{\partial x} - H\Phi + S$$

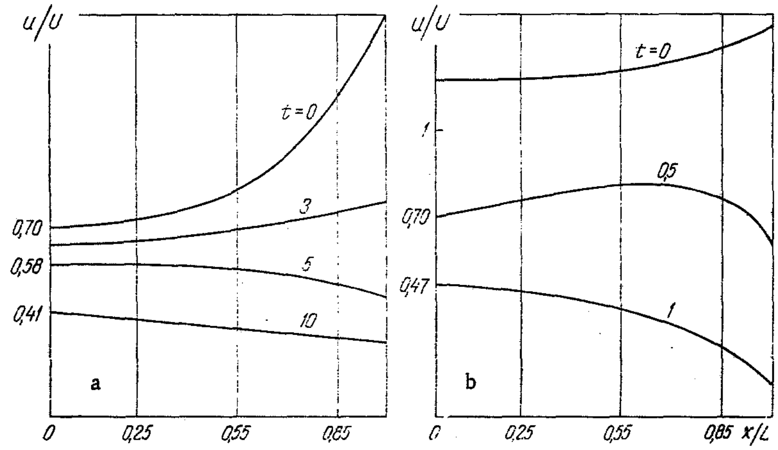

Fig. 2. Variation in velocity along length of gas main at separate times: a) with $\alpha = 0$; b) with $\alpha = \pi/2$.

TABLE 1. Variation in Gas Flow Rate along Length of Gas Main and with Time

t	x: 0	0,25	0,35	0,45	0,55	0,65	0,75	0,85	0,95	1
					$\alpha = 0$					
0	3,402	3,410	3,412	3,416	3,419	3,420	3,420	3,415	3,401	3,350
1	3,503	3,495	3,490	3,484	3,475	3,462	3,444	3,417	3,369	3,250
2	3,394	3,385	3,376	3,363	3,346	3,324	3,295	3,256	3,204	3,130
3	3,181	3,172	3,162	3,149	3,132	3,119	3,080	3,044	2,999	2,940
4	2,935	2,927	2,919	2,908	2,893	2,874	2,850	2,821	2,788	2,740
5	2,699	2,693	2,687	2,678	2,666	2,651	2,632	2,689	2,583	2,550
6	2,489	2,484	2,479	2,471	2,462	2,450	2,436	2,418	2,398	2,370
7	2,306	2,302	2,298	2,292	2,285	2,276	2,265	2,252	2,236	2,211
8	2,153	2,150	2,147	2,142	2,136	2,129	2,121	2,111	2,099	2,080
9	2,090	2,027	2,025	2,022	2,017	2,012	2,006	1,998	1,990	1,970
10	1,938	1,936	1,935	1,932	1,929	1,925	1,921	1,916	1,910	1,901
					$\alpha = \pi/2$					
0	9,337	7,342	6,472	5,680	4,961	4,311	3,724	3,196	2,721	2,290
0,5	5,761	5,686	5,596	5,464	5,281	5,036	4,717	4,309	3,796	3,161
1	3,797	3,701	3,581	3,406	3,173	2,877	2,616	2,087	1,587	1,011

was examined and the method and system for its numerical solution were indicated. In comparing (2.7) with this equation we note that the system for the solution of the equation presented can be applied to (2.7).

The problem is solved by the trial run method. A program has been compiled on the Razdan-3 computer for this purpose.

3. Numerical Example

Let us present a concrete numerical example for the cases of $\alpha = 0$ (horizontal gas main) and $\alpha = \pi/2$ (vertical gas main). For the solution of the problem we assign the following numerical data:

$$L = 3\cdot 10^3\,\text{m};\ d = 0.7\,\text{m};\ T_s = 360\,°\text{K};\ p_s = 25\cdot 10^4\ \text{kg/m}^2;$$

$$T_e = 240\,°\text{K};\ \lambda = 0.012;\ p_e = 5\cdot 10^4\ \text{kg/m}^2;$$

$$R = 50\ \text{kg-m/kg}\cdot\text{deg};\ G_0(0) = 99\ \text{kg/sec};\ G_0\left(\frac{\pi}{2}\right) = 57\ \text{kg/sec};$$

$$P_0(x,\ 0) = 1.74(1-0.3\,x)^2 - 0.74;\ P_0\left(x,\ \frac{\pi}{2}\right) = -0.02(1-0.3\,x)^2 +$$

$$+ 1.02(1-0.3\,x)^{9.8};\ G(t) = G_0\left(1 - \frac{1}{2}\ \sin 0.13t\right),$$

where G_0 is the average hourly gas flow rate and t is the time.

A four-point implicit system is used and the practical convergence of the method is assured.

The distributions of pressures and velocities along the length of the gas main at separate times are presented in Figs. 1 and 2. In comparing the results obtained we note that with an increase in the slope the pressure increases considerably while the velocity decreases. For example, at the time t = 1 the pressure at the middle of the pipeline increases by 1.2 times and at the end by 2 times. These changes increase even more with time within the limits of the period of rupture of the gas main.

From Fig. 2 we note that in this case, in contrast to the isothermal mode of motion, the velocity curves, starting with a certain time, change the direction of concavity, and the velocities for fixed values decrease along the length of the gas main.

With an increase in the slope of the gas main in the positive direction the velocities decrease considerably, while in the opposite case, conversely, they increase. In comparing the results of the numerical calculation for the cases of $\alpha = 0$ and $\alpha = \pi/2$ we note that the velocity decreases by 1.5-2 times at separate times in separate pipeline cross sections. In this case in the initial cross sections, as it was assumed, the disparities are not very great, while in subsequent cross sections they increase with the passage of time.

The variations in the gas flow rate along the length of the pipeline and with time as well as the results of the calculation with $\alpha = 0$ and $\alpha = \pi/2$ are presented in Table 1. In comparing these results we note that there is considerable quantitative disparity between them.

Thus, the free convection term plays an important role in the equations of motion with nonisothermal unsteady gas movement through a sufficiently long gas main of large diameter. Consequently, in the planning of long gas mains in localities crossed by mountains it is necessary to allow for the variation in slope of the gas main.

NOTATION

P, pressure; x, coordinate; λ, coefficient of hydraulic resistance; ρ, density; u, velocity; δ, hydraulic radius; g, acceleration of gravity; α, slope of gas main; t, time; T, absolute temperature; R, gas constant; G, flow rate; s, cross-sectional area.

LITERATURE CITED

1. I. A. Charnyi, Unsteady Motion of a Real Fluid in Pipes [in Russian], Gostekhizdat, Leningrad (1951).
2. A. Sh. Asaturyan, Z. T. Galiullin, and V. I. Chernikin, Izv. Vyssh. Uchebn. Zaved., Neft' Gaz, No. 10 (1961).
3. G. A. Babadzhanyan, Izv. Vyssh. Uchebn. Zaved., Neft' Gaz, No. 1 (1961).
4. Z. T. Galiullin, Tr. Vses. Nauchn.-Issled. Inst. Gaza, No. 29/37 (1967).
5. B. L. Krivoshein, E. M. Minskii, V. P. Radchenko, I. E. Khodonovich, and M. G. Khublaryan, Inzh.-Fiz. Zh., 13, No. 4 (1967).
6. Z. T. Galiullin and V. I. Chernikin, Izv. Vyssh. Uchebn. Zaved., Neft' Gaz, No. 12 (1960).
7. E. I. Minskii, Yu. I. Maksimov, and A. S. Malykh, Tr. Vses. Nauchn.-Issled. Inst. Gaza, No. 13/21 (1961).
8. I. E. Khodonovich and F. G. Tempel', Tr. Vses. Nauchn.-Issled. Inst. Gaza, No. 13/21 (1961).
9. P. I. Tugunov, Author's Abstract of Doctoral Dissertation, Moscow (1970).
10. I. G. Gorokhova, Izv. Vyssh. Uchebn. Zaved., Neft' Gaz, No. 5 (1973).
11. G. A. Babadzhanyan, Izv. Akad. Nauk ArmSSR, Mekh., 27, No. 3 (1974).
12. G. A. Babadzhanyan and L. E. Danielyan, Izv. ArmSSR, Mekh., 24, No. 6 (1971).
13. L. E. Danielyan, Izv. Akad. Nauk ArmSSR, Mekh., 23, No. 1 (1970).
14. I. S. Berezin and N. P. Zhidkov, Methods of Calculations [in Russian], Vol. 2, Fizmatgiz, Moscow (1960).
15. R. D. Richtmyer, Difference Methods for Initial-Value Problems, 2nd ed., Wiley (1967).
16. A. S. Smirnov, L. A. Genkina, M. M. Khushpulyan, and D. L. Chernov, The Transport and Storage of Gas [in Russian], Moscow (1962).